Section 2.3 – Long/Synthetic Division and the Remainder & Factor Theorems

Long Division of Polynomials

Polynomial Long Division is a lot like dividing integers (you should all be experts at this! ☺)

$$32\overline{)3587}$$

Now let's try it with polynomials instead of ordinary numbers:

Ex.1)

$$x-2\overline{)x^3+3x^2-4x-12}$$

Ex.2) Divide $2x^4+3x^3+5x-1$ by x^2-2x+2 (careful setting this one up!)

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Synthetic Division of Polynomials

Synthetic division can be used to divide a polynomial by an expression of the form x-k.

Let's do Example 1 again, but this time with synthetic division

$$x-2\overline{\smash{)}\,x^3+3x^2-4x-12}$$

In synthetic division, you don't write the variables.

Step 1: Write the coefficients of the polynomial and then write the k–value (2) of the divisor $x-2$ on the left. Write the 1st coefficient 1 below the line.

$$\begin{array}{r|rrrr} 2 & 1 & 3 & -4 & -12 \\ & \downarrow & & & \\ \hline & 1 & & & \end{array}$$

Step 2: Multiply the k–value (2) by the number below the line and write the product below the next coefficient.

$$\begin{array}{r|rrrr} 2 & 1 & 3 & -4 & -12 \\ & & 2 & & \\ \hline & 1 & & & \end{array}$$

Step 3: Write the <u>sum</u> of 3 and 2 below the line. Multiply 2 by the number below the line and write the product below the next coefficient.

$$\begin{array}{r|rrrr} 2 & 1 & 3 & -4 & -12 \\ & & 2 & 10 & \\ \hline & 1 & 5 & & \end{array}$$

Step 4: Write the sum of -4 and 10 below the line. Multiply 2 by the number below the line and write the product below the next coefficient.

$$\begin{array}{r|rrrr} 2 & 1 & 3 & -4 & -12 \\ & & 2 & 10 & 12 \\ \hline & 1 & 5 & 6 & 0 \end{array} \leftarrow \text{ remainder}$$

The remainder is 0, and the resulting numbers 1, 5, and 6 are the coefficients of the quotient

So your answer is...... x^2+5x+6 Just like with long division!

Let's try some more:

Ex 3) $(5x^4-2x^3+7x^2+6x-8)\div(x-4)$

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Ex 4) Divide $x^3 - 10x - 24$ by $x + 2$ (be careful…)

Ex 5) $(2x^3 - 3x + 4) \div (x - 1)$

The Remainder Theorem

An important case of the division algorithm occurs when the divisor is of the form $d(x) = x - k$.

Division Algorithm:

$$\underset{\text{dividend}}{\underset{\uparrow}{f(x)}} = \underset{\text{divisor}}{\underset{\uparrow}{(x-k)}} \bullet \underset{\text{quotient}}{\underset{\uparrow}{q(x)}} + \underset{\text{remainder}}{\underset{\uparrow}{r}}$$

When $x = k$, what does $f(x)$ equal?

Ex 6) Let $f(x) = 3x^3 - 2x^2 + 2x - 5$

Divide $f(x)$ by $x - 2$ using synthetic division. What is the remainder? How is $f(2)$ related to the remainder?

> **<u>The Remainder Theorem:</u>**
>
> If a polynomial $f(x)$ is divided by $x - k$, then the remainder is $r = f(k)$.

Ex 7) Find the <u>remainder</u> when $f(x) = 3x^2 + 7x - 20$ is divided by:

a) $x - 2$

b) $x + 4$

From the previous page:

Since the remainder in part (b) is 0, $x+4$ **divides evenly** into $f(x)=3x^2+7x-20$.

And therefore, we know:

- $x+4$ is a FACTOR of $f(x)=3x^2+7x-20$,
- -4 is a ZERO of $3x^2+7x-20=0$, and
- $(-4,0)$ is an X-INTERCEPT of the graph of $y=3x^2+7x-20$.

We know all of this without ever dividing, factoring, or graphing.

<u>The Factor Theorem:</u>

A polynomial $f(x)$ has a factor $x-k$ if $f(k)=0$.

Ex 8) Factor $f(x)=2x^3+11x^2+18x+9$ given that $f(-3)=0$.

Because $f(-3)=0$, we know that ______________ is a factor of $f(x)$.
Use synthetic division to simplify the polynomial and find the other factors.

Ex 9) One zero of $f(x)=x^3-2x^2-9x+18$ is $x=2$. Find the other zeros of the function.

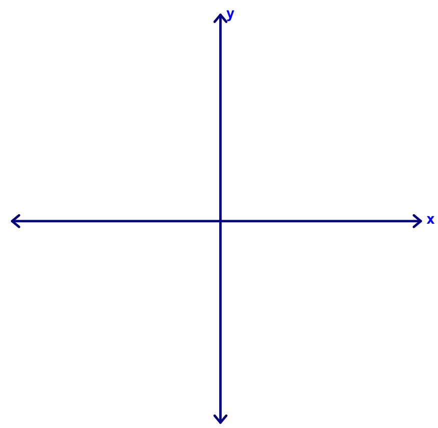
Because $f(2)=0$, you know that _________ is a factor of $f(x)$.
Again, use synthetic division to factor completely.

Ex 10) Use $f(x)=x^3-4x^2-2x+8$ to answer the following questions:

a) Use the zero feature on your calculator to approximate the zeros of the function.

My zeros are approximately _________, ________, and ___________

b) Sketch:



c) Determine one of the exact zeros.

One of the exact zeros is __________

d) Use synthetic division to verify your result, and then factor the polynomial completely.

Homework: Page 159-160 #7, 9, 15, 21, 23, 27, 29, 35, 45(a&b), 49, 51, 65